PHILOSOPHY IN QUEST OF GEOMETRIC RIGOUR

THE ELEMENTS OF PROCLUS AND ITS ORIGINS

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1. Introduction

T 1 Simon Grynaeus (1531): Est autem haud usquam vel scrupulosus Aristoteles vel iniucundus magis quam de his rebus disserendo. itaque magno taedio Proclus Aristotelicae philosophiae studiosos levabit.

T 2 Patrizi (1583): Verum ad Proclum redeamus. cuius elementa Theologica tibi Parolari eruditissime animo libenti damus. Uti cognoscas, quanto ingenii acumine mathematico more, Theologia Platonis universa mira demonstrationum necessitate sit demonstrata. addita sunt physica quoque elementa, ex Aristotelis dogmatibus desumpta, eodemque modo tradita: quo modo, res nulla alia quam mathematica, a quoquam unquam alio, est tractata: et quae suo hoc exemplo sublimium philosophorum ingenia in reliquis philosophiae partibus movere queat ad imitandum.

T 3 (title) Νικολάου Ἐπισκόπου Μεθώνης Ἀνάπτυξις τῆς Θεολογικῆς Στοιχειώσεως Πρόκλου τοῦ Λυκίου πλατωνικοῦ φιλοσόφου πρὸς τὸ μὴ συναρπάζεσθαι τοὺς ἀναγινώσκοντας ὑπὸ τῆς ὑποφαινομένης αὐτῇ πειθανάγκης καὶ σκανδαλίζεσθαι κατὰ τῆς ἀληθοῦς πίστεως.
Explication of the *Elements of Theology* of Proclus of Lycia the Platonic Philosopher: that those who read this book might not be seized by its seemingly compelling persuasion ad be tempted against the true faith. (trans. Joshua Robinson)

2. Formal characteristics of the *mos geometricus*

T 4 Procl., *in Eucl.* 255.8-12 (trans. G. R. Morrow): “Every reduction to impossibility takes the contradictory of what it intends to prove and from this as a hypothesis proceeds until it encounters something admitted to be

absurd and, by thus destroying its hypothesis, confirms the proposition it set out to establish."

3. Proclus' Στοιχείωσις φυσική

4. Proclus' Στοιχείωσις θεολογική

5. The Aristotelian background

T 5 *Phys.* VI.1, 231a20-b18 [tr. ROT, R. P. Hardie and R. K. Gaye, rev. J. Barnes]

Ζ.

Εἰ δ' ἐστὶ συνεχὲς καὶ ἁπτόμενον καὶ ἐφεξῆς, ὡς διώρισται πρότερον, συνεχῆ μὲν ὧν τὰ ἔσχατα ἕν, ἁπτόμενα δ' ὧν ἅμα, ἐφεξῆς δ' ὧν μηδὲν μεταξὺ συγγενές, ἀδύνατον ἐξ ἀδιαιρέτων εἶναί τι συνεχές, οἷον γραμμὴν ἐκ στιγμῶν, εἴπερ ἡ γραμμὴ μὲν συνεχές, ἡ στιγμὴ δὲ ἀδιαίρετον. οὔτε γὰρ ἓν τὰ ἔσχατα τῶν στιγμῶν (οὐ γάρ ἐστι τὸ μὲν ἔσχατον τὸ δ' ἄλλο τι μόριον τοῦ ἀδιαιρέτου), οὔθ' ἅμα τὰ ἔσχατα (οὐ γάρ ἐστιν ἔσχατον τοῦ ἀμεροῦς οὐδέν· ἕτερον γὰρ τὸ ἔσχατον καὶ οὗ ἔσχατον). ἔτι δ' ἀνάγκη ἤτοι συνεχεῖς εἶναι τὰς στιγμὰς ἢ ἁπτομένας ἀλλήλων, ἐξ ὧν ἐστι τὸ συνεχές· ὁ δ' αὐτὸς λόγος καὶ ἐπὶ πάντων τῶν ἀδιαιρέτων. συνεχεῖς μὲν δὴ οὐκ ἂν εἶεν διὰ τὸν εἰρημένον λόγον· ἅπτεται δ' ἅπαν ἢ ὅλον ὅλου ἢ μέρος μέρους ἢ ὅλου μέρος. ἐπεὶ δ' ἀμερὲς τὸ ἀδιαίρετον, ἀνάγκη ὅλον ὅλου ἅπτεσθαι. ὅλον δ' ὅλου ἁπτόμενον οὐκ ἔσται συνεχές. τὸ γὰρ συνεχὲς ἔχει τὸ μὲν ἄλλο τὸ δ' ἄλλο μέρος, καὶ διαιρεῖται εἰς οὕτως ἕτερα καὶ τόπῳ κεχωρισμένα.

b6 ἀλλὰ μὴν οὐδὲ ἐφεξῆς ἔσται στιγμὴ στιγμῇ ἢ τὸ νῦν τῷ νῦν, ὥστ' ἐκ τούτων εἶναι τὸ μῆκος ἢ τὸν χρόνον· ἐφεξῆς μὲν γάρ ἐστιν ὧν μηθέν ἐστι μεταξὺ συγγενές, στιγμῶν δ' αἰεὶ μεταξὺ γραμμὴ καὶ τῶν νῦν χρόνος.

ἔτι διαιροῖτ' ἂν εἰς ἀδιαίρετα, εἴπερ ἐξ ὧν ἐστιν ἑκάτερον, εἰς ταῦτα διαιρεῖται· ἀλλ' οὐθὲν ἦν τῶν συνεχῶν εἰς ἀμερῆ διαιρετόν. ἄλλο δὲ γένος οὐχ οἷόν τ' εἶναι μεταξύ. ἢ γὰρ ἀδιαίρετον ἔσται ἢ διαιρετόν, καὶ εἰ διαιρετόν, ἢ εἰς ἀδιαίρετα ἢ εἰς ἀεὶ διαιρετά· τοῦτο δὲ συνεχές.

b15 φανερὸν δὲ καὶ ὅτι πᾶν συνεχὲς διαιρετὸν εἰς αἰεὶ διαιρετά· εἰ γὰρ εἰς ἀδιαίρετα, ἔσται ἀδιαίρετον ἀδιαιρέτου ἁπτόμενον· ἓν γὰρ τὸ ἔσχατον καὶ ἅπτεται τῶν συνεχῶν.

[**Definitions**] Now if the terms 'continuous', 'in contact', and 'in succession' are understood as defined above—[Def.$_A$ 1] things being continuous if their extremities are one, [Def.$_A$ 2] in contact if their extremities are together, and [Def.$_A$ 3] in succession if there is nothing of their own kind intermediate between them—

[**Theorem$_A$ 1**] nothing that is continuous can be composed of indivisibles: e.g. a line cannot be composed of points, the line being continuous and the point indivisible.

[Proof 1] For the extremities of two points can neither be one (since of an indivisible there can be no extremity as distinct from some other part) nor together (since that which has no parts can have no extremity, the extremity and the thing of which it is the extremity being distinct).

[Proof 2 by reductio] Moreover, if that which is continuous is composed of points, these points must be either continuous or in contact with one another: and the same reasoning applies in the case of all indivisibles.

Now for the reason given above [cf Proof 1] they cannot be continuous; and one thing can be in contact with another only if whole is in contact with whole or part with part or part with whole. But since indivisibles have no parts, they must be in contact with one another as whole with whole. And if they are in contact with one another as whole with whole, they will not be continuous; for that which is continuous has distinct parts, and these parts into which it is divisible are different in this way, i.e. spatially separate.

[**Theorem A 2**] Nor, again, can a point be in succession to a point or a now to a now in such a way that length can be composed of points or time of nows;

[Proof] for things are in succession if there is nothing of their own kind intermediate between them [cf Def. 1], whereas intermediate between points there is always a line and between nows a period of time.

[Reductio] Again, they could be divided into indivisibles, since each is divisible into the parts of which it is composed. But, as we saw [cf. Theorem A 1], no continuous thing is divisible into things without parts. Nor can there be anything of any other kind between; for it would be either indivisible or divisible, and if it is divisible, divisible either into indivisibles or into divisibles that are always divisible, in which case it is continuous.

[**Theorem A 3**] Moreover, it is plain that everything continuous is divisible into divisibles that are always divisible;

[Proof by reductio] for if it were divisible into indivisibles, we should have an indivisible in contact with an indivisible, since the extremities of things that are continuous with one another are one and are in contact [cf Def. 2].

T 6 *Phys.* VI.1, 231a21 slightly rewritten

Ὅροι

1. συνεχῆ ὧν τὰ ἔσχατα ἕν.
2. ἁπτόμενα ὧν ἅμα τὰ ἔσχατα
3. ἐφεξῆς ὧν μηδὲν μεταξὺ συγγενές

[Propositiones, Θεωρήματα]

1. ἀδύνατον ἐξ ἀδιαιρέτων εἶναί τι συνεχές. [...]
2. οὐκ ἐφεξῆς ἔσται στιγμὴ στιγμῇ ἢ τὸ νῦν τῷ νῦν, ὥστ' ἐκ τούτων εἶναι τὸ μῆκος ἢ τὸν χρόνον· [...]
3. πᾶν συνεχὲς διαιρετὸν εἰς αἰεὶ διαιρετά· [...]

T 7 Proclus *El. phys.*, I

Ὅροι τοῦ πρώτου. Definitions of the first book

1. Συνεχῆ ἐστιν, ὧν τὰ πέρατα ἕν. (= Def. A 1)

Continuous are [things] whose limits are one.

2. Ἁπτόμενά ἐστιν, ὧν τὰ πέρατα ἅμα. (= Def. A 2)

Touching are [things] whose limits are together.

3. Ἐφεξῆς ἐστιν, ὧν μηδὲν μεταξὺ ὁμογενές. (= Def. A 3)

Successive are those [things] between which there is nothing of the same kind.

[...]

THEOREMS

1. Δύο ἀμερῆ οὐχ ἅψεται ἀλλήλων. Two partless [things] will not touch one another. [used in the proof of Theor. A 3]

[...]

2. Δύο ἀμερῆ συνεχὲς οὐδὲν ποιήσει. Two partless [things] will not produce anything continuous. [= **Theor. A 1**]

> Εἰ γὰρ δυνατόν, ἔστω δύο ἀμερῆ τὰ AB καὶ ποιείτω συν-εχὲς τὸ ἐξ ἀμφοῖν. ἀλλὰ πάντα τὰ συνεχῆ ἅπτεται πρότερον· τὰ ἄρα AB ἅπτεται ἀλλήλων ἀμερῆ ὄντα, ὅπερ ἀδύνατον. For if this is possible, let there be two partless [things] A and B and let them produce as continuous that which consists of both of them. However, all continuous things first touch. Thus, A and B touch one another while being partless, which is impossible.
>
> Ἄλλως. Εἰ ἔστι συνεχὲς ἐκ τῶν AB ἀμερῶν, ἢ ὅλον ὅλου ἅπτεται τὸ A τοῦ B ἢ ὅλον μέρους ἢ μέρει μέρους. ἀλλ' εἰ μὲν ὅλον μέρους ἢ μέρει μέρους, οὐκ

ἔσται ἀμερῆ τὰ AB, εἰ δὲ ὅλον ὅλου ἅπτοιτο, οὐκ ἔσται συνεχές, ἀλλ' ἐφαρμόσει μόνον· εἰ οὖν οὐκ ἦν τὸ A συνεχὲς, οὐδὲ τὸ B μετὰ τοῦ A ἔσται συνεχὲς ὅλον ὅλου ἁπτόμενον.

In another manner. If there is [something] continuous [composed] of the partless [things] A and B, either A touches B as one whole another or as a whole a part or it touches a part with a part. If, however, a whole touches a part or it touches a part with a part, A and B will not be partless. But if a whole would touch a whole, it will not be continuous, but they merely coincide; if then A were not continuous, B will not be continuous together with A either, when as a whole it touches [A] as a whole.

3. Τῶν ἐν συνεχεῖ ἀμερῶν τὸ μεταξὺ συνεχές. What is between partless [things that are] in something continuous is continuous. [used in the proof of Theor. A 2]

[...]

4. Δύο ἀμερῆ ἐφεξῆς ἀλλήλοις οὐκ ἔστιν. Two partless [things] are not successive to one another [= **Theor. A 2**]

Ἔστω γὰρ δύο ἀμερῆ τὰ AB· λέγω ὅτι οὐκ ἔσται ἐφεξῆς τὸ A τῷ B. ἐπεὶ γὰρ δέδεικται, ὅτι δύο ἀμερῶν τὸ μεταξὺ συνεχές ἐστιν, ἔστω δὴ τὸ μεταξὺ αὐτῶν τὸ ΓΔ καὶ διῃρήσθω κατὰ τὸ E· τὸ E ἄρα ἀμερές ἐστι μεταξὺ ὂν τῶν AB· ἐφεξῆς δὲ ἦν, ὧν μηδὲν μεταξὺ ὁμογενές· οὐκ ἄρα τὸ A καὶ τὸ B ἐφεξῆς ἐστιν.

A_____ B
Γ E Δ

For let there be two partless [things] A and B. I say that A will not be continuous with B. For since it has been shown that what lies between two partless [things] is continuous, let what lies between them be [called] CD and let it be divided at E. Thus, E is partless, lying between A and B. Successive were those [things] between which there is nothing of the same kind. A and B then are not successive.

5. Πᾶν συνεχὲς διαιρετόν ἐστιν εἰς ἀεὶ διαιρετά.

Everything continuous is divisible into [parts] that are always divisible. [= **Theor. A 3**]

T 8 *Phys.* VI.1, 231b21-25: εἰ γὰρ τὸ μέγεθος ἐξ ἀδιαιρέτων σύγκειται, καὶ ἡ κίνησις ἡ τούτου ἐξ ἴσων κινήσεων ἔσται ἀδιαιρέτων, οἷον εἰ τὸ ΑΒΓ ἐκ τῶν Α Β Γ ἐστὶν ἀδιαιρέτων, ἡ κίνησις ἐφ' ἧς ΔΕΖ, ἣν ἐκινήθη τὸ Ω ἐπὶ τῆς ΑΒΓ, ἕκαστον τὸ μέρος ἔχει ἀδιαίρετον.

T 9 Thom. Aq. *in Phys.*, lib. V lect. v, 684: *Praemittit aetem haec, quia horum definitionibus utitur in demonstrationibus consequentibus per totum librum; sicut et in principio Euclidis ponuntur definitiones, quae sunt sequentium demonstrationum principia.*

T 10 Autolycus, *De sphaera quae movetur*

[Ὅροι]

1. Ὁμαλῶς λέγεται φέρεσθαι σημεῖα ὅταν ἐν ἴσῳ χρόνῳ ἴσα τε ᾖ καὶ ὅμοια μεγέθη διεξέρχηται·

2. ἐὰν δὲ ἐπί τινος γραμμῆς φερόμενόν τι σημεῖον ὁμαλῶς δύο γραμμὰς διεξέλθῃ, τὸν αὐτὸν ἕξει λόγον ὅ τε χρόνος πρὸς τὸν χρόνον ἐν ᾧ τὸ σημεῖον ἑκατέραν τῶν γραμμῶν διεξῆλθεν καὶ ἡ γραμμὴ πρὸς τὴν γραμμήν.

[Προτάσεις]

1. Ἐὰν σφαῖρα στρέφηται ὁμαλῶς περὶ τὸν ἑαυτῆς ἄξονα, πάντα τὰ ἐπὶ τῆς ἐπιφανείας τῆς σφαίρας σημεῖα ὅσα μὴ ἔστιν ἐπὶ τοῦ ἄξονος κύκλους γράψει παραλλήλους τοὺς αὐτοὺς πόλους ἔχοντας τῇ σφαίρᾳ, καὶ ἔτι ὀρθοὺς πρὸς τὸν ἄξονα.

Ἔστω σφαῖρα ἧς ἄξων ἔστω ἡ αβ′ εὐθεῖα, πόλοι δὲ αὐτῆς τὰ α′ β′ σημεῖα, καὶ στρεφέσθω ὁμαλῶς περὶ τὸν ἑαυτῆς ἄξονα τὸν αβ′· λέγω ὅτι κτλ.

6. Deviating from the *mos geometricus* in the *Elements of Theology*

T 11 Eucl., *El.* 1 C.N. 8: Καὶ τὸ ὅλον τοῦ μέρους μεῖζον.
And the whole is bigger than the part.

T 12 *in Parm.* VI, 1091.19-23: νῦν οὖν ληπτέον ὡς ὁ Πλάτων παντὸς τοῦ τῶν ἑνάδων πλήθους ἐξαίρει τὸ ἓν ὡς γεννητικὸν αὐτοῦ καὶ ὑποστατικόν, καὶ τοῦτο ποιεῖ κατὰ τὰς κοινὰς ἡμῶν ἐννοίας προλαβὼν ὡς *οὐ πολλὰ τὸ ἕν* (*Parm.* 137c4-5: εἶεν δή, φάναι· εἰ ἕν ἐστιν, ἄλλο τι οὐκ ἂν εἴη πολλὰ τὸ ἕν;)
For the present, then, let us take it that Plato removes the One from the whole multiplicity of henads as generating them and bringing them to existence, and he does this by taking as a first premise, following the common concepts established within us, that the One is not Many. (trans. Dillon - Morrow)

T 13 *in Parm.* VI, 1092.15-27: ἐκεῖνο οὖν πρῶτον ῥητέον *ὅτι τὸ μὴ πολλὰ εἶναι τὸ ἓν* οὐδὲ ἀποδείξεως οὐδὲ παραμυθίας ἠξίωσεν, ἀλλὰ κατὰ τὴν κοινὴν ἔλαβεν αὐτὴν καὶ ἀδιάστροφον ἔννοιαν· δεῖ γὰρ ἐν ταῖς περὶ τοῦ πρώτου μάλιστα θεωρίαις ἀνεγεῖρειν τὰς κοινὰς αἰτίας, ἐπειδὴ πάντα αὐτοφυῶς εἰς ἐκεῖνο τέτακται καὶ ἀνεπιτηδεύτως καὶ ὅσα κατὰ νοῦν ἐνεργεῖ καὶ ὅσα κατὰ φύσιν μόνον. καὶ ὅλως ἁπάσης ἀποδείξεως ἀρχὴν εἶναι τὸ ἀναπόδεικτον καὶ τὰς κοινὰς ἐννοίας προηγεῖσθαι τῶν ἀποδείξεων, ὥσπερ καὶ γεωμέτραι φασί. τοῦ δὲ, ὅτι τὸ ἓν *οὐ πολλά*, γνωριμώτερον οὐδέν ἐστιν ἡμῖν καὶ σαφέστερον· ὅθεν τοῦτο οὐ δεηθεὶς κατασκευῆς οὐδὲ πλείονος λόγου παρέλαβεν. εἴληπται μὲν οὖν ἀπὸ τῆς κοινῆς ἐννοίας ὅτι οὐ πολλὰ τὸ ἕν· εἰ δὲ χρή τινος αὐτὸ καὶ παραμυθίας ἀξιῶσαι, λέγωμεν ὅτι κτλ.
The first thing to be said is that Parmenides does not think that the statement that the One is not Many requires either proof or argument to support it, but he has assumed it on the basis of our common and uncorrupted intuition; for in our speculations about the first principle we should especially stimulate the common intuitions, since everything is naturally and unaffectedly related to it, both what operates according to

intellect and what operates only according to nature. And, in general, the beginning of every process of proof must be the indemonstrable, and common intuitions must come before demonstrations, as the geometers also assert. Now nothing is more familiar or obvious to us than the statement that the One is not Many, for which reason he assumed this as not requiring introduction or further argument. It is assumed on the basis of common intuition that the One is not Many; if one is to demand some argument for this also, we may say that etc.

T 14 *in Parm.* VI, 1099.6-9: δύο δὴ τούτων ἡμῖν ἀξιωμάτων ὑποκειμένων, κατίδωμεν ὅπως συλλογίζεται ὁ Παρμενίδης ὅτι *τὸ ἓν οὔτε ὅλον ἐστὶν οὔτε μέρη ἔχει,* <καὶ> παρακολουθήσωμεν αὐτοῦ τῇ ἀγωγῇ.
So then let these two axioms be regarded as established, and let us see how Parmenides concludes on this basis that the One is neither a whole nor has parts, and let us follow his line of reasoning.

T 15 *in Parm.* VI, 1099.29-1100.3: καὶ ὅρα τὴν γεωμετρικὴν τάξιν, ὅπως ὅτι μὲν *οὐ πολλὰ τὸ ἓν* εἴληπται ὡς ἀξίωμα καὶ ὡς κοινὴ ἔννοια, ὅτι δὲ *οὔτε ὅλον ἐστὶν οὔτε μέρη ἔχει* κατασκευάζεται δι' ἐκείνης τῆς κοινῆς ἐννοίας·
And observe the geometrical order in this, how the proposition that the One is not Many has been taken as an axiom and as a common intuition, whereas the proposition that it is neither a whole nor has parts is established by means of that common intuition.

T 16 *ET* 1: Πᾶν πλῆθος μετέχει πῃ τοῦ ἑνός. (translations of *ET* after E.R. Dodds)
Every plurality in some way participates unity / the one.

7. The *Elements of Theology* as a science

8. Bibliography

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